Instability of isolated planar shock waves

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Abstract

Previously, expressions governing the temporal evolution of linear perturbations to an isolated, planar, two-dimensional shock front in an inviscid fluid medium with an arbitrary equation of state were derived using a methodology based on Riemann invariants and Laplace transforms [J.W. Bates, Phys. Rev. E **69**, 056313 (2004)]. An overlooked yet immediate consequence of this theory is that the stability limits of shocks can be readily determined from a inspection of the poles of the transformed ripple amplitude. Here, it is shown that two classes of instabilities exist for isolated planar shock waves: one in which perturbations grow exponentially in time, and the other in which disturbances are stationary. These results agree with those derived by D'yakov and Kontorovich (by more arduous and somewhat ambiguous means), and serve as an important addendum to our earlier analysis.

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I. INTRODUCTION

The stability of shock waves is an issue of fundamental importance in compressible fluid dynamics, with far-reaching implications in fields ranging from astrophysics [1] to inertial confinement fusion [2]. Under most circumstances, a small sinusoidal disturbance to an otherwise planar shock front in a homogeneous fluid will oscillate with an amplitude that decays asymptotically according to an inverse power law (*e.g.*, as $t^{-3/2}$, where t denotes time [3–5]). Even in the absence of viscous damping effects, all linear disturbances to the front will eventually die away, and the shock will regain its planarity. Roberts [6] was the first to show that perturbed shocks in fluids obeying an ideal-gas equation of state (EOS) always display this asymptotic behavior, and are thus unconditionally stable. For a fluid with a non-ideal EOS, however, shock stability is not guaranteed, and under certain conditions, small perturbations actually may amplify rather than evanesce, ultimately leading to a reorganization of the steady-state flow [7–10].

The initial theoretical investigation of this subject was performed over 50 years ago by D'yakov [11], who used a normal-mode analysis to determine conditions in which "corrugations" on an isolated shock traveling through an arbitrary quiescent fluid at constant speed D would grow exponentially in the linear approximation. The use of the term "isolated" here refers to an idealized situation in which a shock is decoupled completely from its driving mechanism (*e.g.*, a moving piston, or an ablation surface). Although derived in a somewhat cryptic fashion with reliance of numerous variable transformations, D'yakov's results proved to be seminal because they stated for the first time shock instability criteria in terms of simple physical quantities that were useful to experimentalists. Using the so-called "D'yakov parameter"

$$h = -\frac{D^2}{\eta^2}\left(\frac{d\rho}{dp}\right)_H , \tag{1}$$

whose derivative is taken along the Hugoniot curve in the pressure-density (p, ρ) plane, and evaluated at the downstream state, these criteria can be summarized as

$$h < -1 \qquad \text{or} \qquad h > 1 + 2M_1 . \tag{2}$$

Here, $M_1 = (D - U)/c_1$ is a downstream Mach number satisfying $0 < M_1 < 1$, where U is the shocked fluid velocity in the laboratory frame, and c_1 is the associated sound speed. The quantity $\eta = \rho_1/\rho_0 > 1$ in Eq. (1) is the compression ratio across the shock, where the

subscripts "0" and "1" denote upstream and downstream states, respectively.

In the 1970s, Swan and Fowles [12, 13] revisited the shock stability problem in an effort to clarify several issues that arose in D'yakov's original analysis. These authors carefully repeated the laborious normal-mode calculation, and found that despite the ambiguities in his derivation, D'yakov's conclusions were nevertheless correct. Furthermore, Swan and Fowles showed that the findings of a separate investigation due to Erpenbeck [14] — whose mathematical approach was different and whose results were cast in terms of thermodynamic quantities — were equivalent to the inequalities appearing in Eq. (2). Using Erpenbeck's formulation of the instability criteria, Gardner [8] was able to show that satisfaction of the condition $1 + 2M_1 < h$ leads to the splitting of a shock into two counter-propagating waves — although apparently this phenomena has never been observed experimentally. The other instability criterion, $h < -1$, is known to correspond to the break-up of a front into two waves traveling in the same direction [7], and has been documented for some time in experiments with shocked materials undergoing phase transformations, or yielding at the elastic limit [15].

In addition to temporally exponentiating solutions, D'yakov's analysis (which contained an error that was later corrected by Kontorovich [16]) also identified a regime in which the amplitude of shock perturbations remains stationary. This phenomenon is known as the "D'yakov-Kontorovich" (DK) [17, 18] or "acoustic emission" [19, 20] instability, although such terminology is arguably misleading since the growth rate is zero for all perturbative modes, and no *true* instability takes place — at least not in the conventional sense. Once created, small disturbances on the front persist indefinitely — without amplifying or damping — and continuously radiate sound and entropy vortex waves into the downstream flow behind the shock. The DK instability can occur when h satisfies

$$h_c \equiv \frac{1 - M_1^2(1+\eta)}{1 - M_1^2(1-\eta)} < h\,. \tag{3}$$

Fowles [21] has shown that this same expression can be derived by determining the conditions for the reflection coefficient of acoustic waves incident on a shock front from the downstream direction to become infinite. Note that the parameter h_c in Eq. (3) always lies between the limits -1 and 1, therefore extending the range of values of h for which "unstable" shock behavior may occur. For an ideal gas, $h_c = (1 - 2M_0)^{-1}$ and $h = -1/M_0^2$ (where $M_0 = D/c_0 > 1$ is the Mach number of the shock in the *upstream* fluid, and c_0 is the speed

of sound there), and so the inequality in Eq. (3) can never be satisfied. There is evidence to suggest, though, that planar shocks in other substances may experience this type of instability. Examples include van der Waals fluids [18, 20], ionizing [22, 23] and dissociating [24, 25] gases, as well as certain metals [26].

In this paper, we present an alternative derivation of the shock instability criteria appearing in Eqs. (2) and (3) that is more tractable than the traditional normal-mode approach. Although our conclusion is not new, the means that we employ to reach it is. Our analysis relies on a methodology based on Riemann invariants and Laplace transforms that was pioneered by Roberts [6], and extended recently [27] to account for a fluid with an arbitrary EOS. The salient aspects of this underlying theory are summarized in Sec. II, although we should remark that in the interest of brevity, most of the mathematical details have been omitted and only expressions directly relevant to our purposes here have been reproduced; consequently, the reader is encouraged to consult Ref. [27] in concert with what follows. One of the principal results of our earlier investigation was the derivation of an equation for the Laplace-transform of the shock-ripple amplitude, and we show in Sec. III how the shock stability limits can be gleaned from an inspection of the poles of this expression using a simple graphical technique. The validity of these limits is verified also in Sec. III by comparison with numerical solutions of the Volterra equation that the ripple amplitude is known to satisfy. Finally, in Sec. IV we summarize the conclusions of this study.

II. A BRIEF SUMMARY OF OUR EARLIER WORK

Consider an isolated, planar, step shock as described in Sec. I that propagates in the negative x-direction, and has a front of infinite extent that lies parallel to the y-axis. In the frame of reference in which the downstream fluid is at rest, the position of the unperturbed shock as a function of time is given by $x + (D - U)t = 0$. We take this to be the "zeroth-order" state upon which a single-mode transverse perturbation is imposed. The position of the *perturbed* shock can be written as

$$x_s(t) = -(D - U)t + \delta x(t)\, e^{iky}\,, \tag{4}$$

where $i = \sqrt{-1}$, and the real part of the right hand side of this equation is implied. Additionally, the wavenumber k and amplitude δx satisfy the linearity condition $k\,\delta x \ll 1$. Similar

Fourier expansions hold for the perturbed hydrodynamic quantities behind the shock front. Note that multi-mode perturbations can be treated by the inclusion of additional terms in Eq. (4), but owing to the linear independence of the complex exponential functions and the linear approximation made here, one can show that doing so does not affect our final results. For simplicity, we therefore restrict our discussion to a single wavenumber only. Also note that we do not assume that the amplitude $\delta x(t)$ in Eq. (4) has the normal-mode form $\exp(i\omega t)$, with the complex frequency ω satisfying a dispersion relation. While it has been shown that a normal-mode analysis can be used to determine accurately the limits for shock instability [11, 13], it is also a fairly arduous and complicated approach to finding a solution to this problem [19]. Moreover, the normal-mode formulation fails to capture the correct temporal evolution in the stable regime, since the decay of perturbations on the shock front is not simply a damped exponential function of time there. Our strategy for analyzing a perturbed isolated shock follows a different path. Namely, we seek a general solution for $\delta x(t)$ — or more accurately its Laplace transform — that is valid under all circumstances. Such a function contains complete information about the evolution of the rippled shock front, and once derived, can be readily examined to determine the criteria for unstable behavior.

Before proceeding further with the summary of our earlier theory, it is convenient at this stage to introduce several dimensionless expressions. The first is the non-dimensional ripple amplitude $g(\tau)$, which we write as

$$g(\tau) = k(\eta - 1)\delta x/\eta \,, \tag{5}$$

where $\tau = Dkt/\eta$ is a dimensionless time. Next, we define the additional quantities

$$\alpha^2 \;=\; \frac{1 - M_1^2}{M_1^2} \,, \tag{6}$$

$$\beta \;=\; \frac{1 - h}{2M_1} \,, \tag{7}$$

$$\Gamma \;=\; \frac{(1 + h)\eta}{2M_1} \,. \tag{8}$$

Note that in terms of these parameters, the criteria $h < -1$ and $h > 1 + 2M_1$ can be written as $\Gamma < 0$ and $\beta < -1$, respectively. Moreover, the condition $h > h_c$ is tantamount to $\Gamma > \alpha^2\beta$. A summary of the instability limits for isolated, planar, two-dimensional shock waves as functions of β, h, and Γ is depicted graphically in Fig. 1. Throughout the remainder

of this paper, our preference will be to formulate the perturbed shock problem in terms of α, β, and Γ, instead of the more traditional set of parameters η, M_1, and h.

As discussed in detail in Ref. [27], one can solve for the ripple amplitude $g(\tau)$ by adopting the following procedure. First, the perturbed hydrodynamic variables are substituted into the Euler equations and Rankine-Hugoniot relations and linearized. Then, using a technique based on Riemann invariants, the resulting partial-differential equation can be integrated in time to yield a Volterra equation of the second kind for $g(\tau)$. The result is

$$g(\tau) = F(\tau) + \int_0^\tau g(\theta) K(\tau - \theta) d\theta \,, \tag{9}$$

The kernel $K(q)$ appearing in the expression above is given by

$$K(q) = \frac{\alpha\beta}{1+\beta} \left[J_1(\alpha q) - \frac{\Gamma}{\alpha^2 \beta} \int_0^{\alpha q} J_0(w) dw \right] , \tag{10}$$

where J_0 and J_1 are Bessel functions of order zero and one, respectively. For the case of a planar shock initially deformed into a sinusoidal shape, the function $F(\tau)$ in Eq. (9) assumes a particularly simple form:

$$F(\tau) = g(0) \, \frac{1 + \beta J_0(\alpha\tau)}{1+\beta} \,. \tag{11}$$

Here, the symbol $g(0) = F(0)$ denotes the initial (dimensionless) perturbation amplitude. Taking the Laplace transform of Eq. (9) leads to the expression

$$\frac{g_L(s)}{g(0)} = \frac{\sqrt{s^2+\alpha^2} + \beta s}{s\sqrt{s^2+\alpha^2} + \beta s^2 + \Gamma} \,, \tag{12}$$

where s is the standard Laplace variable, and the notation $g_L(s)$ is a convenient shorthand notation for denoting the transform of the ripple amplitude. As shown in Ref. [27], this function can be manipulated into a sum of pairs of multiplicative terms — each of which is recognizable as the Laplace transform of a known function. Using the convolution theorem, each product then can be inverse transformed to the time domain to yield a final expression for $g(\tau)$.

Following this procedure, it was also demonstrated previously that different families of stable solutions exist, with the envelope of oscillations decaying asymptotically in time as $\tau^{-3/2}$. Although satisfaction of the inequalities $0 < \Gamma < \alpha^2\beta$ was tacitly assumed in order to reach that conclusion, it is important to note that such a restriction is not a requirement of our earlier work. Indeed, the theory appearing in Ref. [27] permits an examination of the nature of the solution for *any* set of parameters α, β, and Γ, and in this way, provides a

means for probing the stability of isolated planar shocks. An immediately apparent approach for conducting such an analysis is through an inspection of the *poles* of Eq. (12) — a topic that is the subject of the next section.

III. POLES OF $g_L(s)$ AND INSTABILITY LIMITS

It is well-known from control theory [28–30] that the poles of a Laplace transform determine the stability of the time-dependent system from which it is derived. This is a remarkable theorem of mathematical physics, for it implies that one does not need to completely solve a dynamical problem in order to ascertain its late-time behavior. Instability is associated with poles that lie in the right half of the complex plane (*i.e.*, have positive real parts), while purely imaginary values imply stationary perturbations. Although overlooked at the time Ref. [27] was published, the stability criteria appearing in Eqs. (2) and (3) can be readily inferred through the application of this fundamental principle to Eq. (12). Here, we demonstrate how this may be accomplished using a simple method of graphical inspection. Additionally, the validity of the shock instability criteria is verified in this section by solving numerically the Volterra equation in Eq. (9) for several examples with different values of α, β, and Γ.

The poles of Eq. (12) occur where its denominator vanishes. Thus, to investigate the stability of $g(\tau)$, we must solve

$$(s - i\alpha)^{1/2}(s + i\alpha)^{1/2} + \beta s + \Gamma/s = 0\,, \tag{13}$$

which has branch points at $s = \pm i\alpha$. There are four possible roots to this equation, which are

$$\pm \left[\frac{\alpha^2 - 2\beta\Gamma \pm \sqrt{\alpha^4 - 4\beta\Gamma\alpha^2 + 4\Gamma^2}}{2(\beta^2 - 1)}\right]^{1/2} .$$

In general, not all of the terms above will be viable solutions to Eq. (13) for a given set of shock parameters α, β, and Γ, and one must always confirm the validity of a particular root through explicit substitution. For example, one can show that no solutions to Eq. (13) exist when the stability criteria $0 < \Gamma < \alpha^2\beta$ are satisfied. This is, of course, consistent with the fact that the damping of perturbations in the stable regime is not an exponential function of time [27]. This subject will be revisited in our presentation of numerical solutions to the Volterra equation that follows. For the moment, however, we turn our attention to

deriving analytically the conditions for which finite roots of Eq. (13) do exist. There are two cases to consider, and we discuss each separately. They are: (A) a root with a positive real part, which implies an exponentially-growing mode; and (B) a root that is purely imaginary, which corresponds to a mode that is stationary.

A. exponentially-growing modes

Given the particular form of Eq. (13), one can show that a root with a positive real part exists if and only if the root itself is purely real. We can then look for solutions of Eq. (13) by plotting $(s^2+\alpha^2)^{1/2}$ and $-\beta s - \Gamma/s$ individually as functions of the real variable s, and determining under what circumstances these two expressions intersect for $s > 0$. Such a plot is shown in Fig. 2. The thick solid line in this figure denotes the function $(s^2+\alpha^2)^{1/2}$, which asymptotes to the linear function s (the straight dashed line in Fig. 2) as $s \to \infty$. The thin solid lines here represent the family of curves given by $-\beta s - \Gamma/s$ for different values of the parameters β and Γ. We see that points of intersection occur in two cases: (i) when $\Gamma < 0$ and $\beta > 0$, and (ii) when $\Gamma > 0$ and $\beta < -1$. Both situations imply a positive real root of Eq. (13), and thus exponentially growing (*i.e.*, unstable) solutions. These inequalities correspond exactly to D'yakov's limits for the absolute instability of isolated planar shock waves stated in Eq. (2).

B. stationary modes

Next, we wish to determine the conditions for stationary perturbations to exist, which requires that we look for purely imaginary roots of Eq. (13). To do this, it is useful to make the substitution $s = \pm i\tilde{s}$, where the variable $\tilde{s}$ is purely real. Equation (13) then becomes

$$\left(\tilde{s}^{\,2} - \alpha^2\right)^{1/2} = -\beta\tilde{s} + \Gamma/\tilde{s}\,. \tag{14}$$

Note that in arriving at this expression, we have used the relation $(\pm ia - ib)^{1/2}\,(\pm ia + ib)^{1/2} = \pm i(a^{\,2} - b^2)^{1/2}$, where a and b are arbitrary, but real variables. The left and right sides of Eq. (14) are plotted in Fig. 3. The thick solid line denotes the function $(\tilde{s}^{\,2} - \alpha^2)^{1/2}$, whose value is not real for values of $\tilde{s}$ less than α, and asymptotically approaches the linear function $\tilde{s}$ (the straight dashed line in Fig. 3) as $\tilde{s} \to \infty$. The thin solid lines in this figure represent

the family of curves given by $-\beta\tilde{s} + \Gamma/\tilde{s}$ for different values of the parameters β and Γ. A point of intersection of these two functions can occur only under a particular circumstance: namely, when $\beta > -1$ and $-\beta\alpha + \Gamma/\alpha > 0$, or equivalently $\Gamma/\alpha^2 > \beta > -1$. Consulting Fig. 1, we see that these inequalities correspond exactly to the well-known criteria for the DK instability. Such a solution implies the existence of a purely imaginary pole of Eq. (12), and thus oscillatory, undamped solutions. We should remark that the same criterion has been derived previously by Wouchuk and Cavada [20] as part of their investigation of the unstable eigenmode spectrum of shocks launched from rippled pistons. In that study, the authors employed a graphical technique similar to the one adopted here to analyze the stability properties of Zaidel's Bessel-series expansion for the downstream pressure field [31].

C. numerical solutions of the Volterra equation

Figure 4 shows five examples of the evolution of the amplitude of a linear sinusoidal perturbation on a shock front obtained by numerically solving the Volterra equation in Eq. (9) for different values of α^2, β, and Γ [32]. In performing these calculations, one must ensure that the algorithm employed to compute the values of Bessel functions of large argument does so with sufficient accuracy. The curves in Fig. 4 are labeled "a" through "e," and the parameters for each case are listed in Table I, along with the corresponding values of the poles of Eq. (12). Note that the curves labeled "a" and "b" satisfy the inequalities $\beta < -1$ and $\Gamma < 0$, respectively, and are clearly unstable in an absolute sense. Accordingly, both have a pole that is real and positive. Each of the curves labeled "c" and "d" satisfies $\Gamma/\alpha^2 > \beta > -1$, and each exhibits a stationary instability with perturbations that persist indefinitely; also note that both of these curves have purely imaginary poles, as well as a negative real one, corresponding to an exponentially-damped transient solution. Curve "e" is an example of a stable case; it has no poles and a perturbation amplitude that damps asymptotically as $\tau^{-3/2}$.

IV. CONCLUSIONS

The present work is an outgrowth of a previous theoretical investigation [27] on the effect of small disturbances to an isolated, planar, two-dimensional shock wave propagating

through a fluid with an arbitrary EOS. In that study, an explicit expression for the time-dependent Fourier coefficient associated with a linear single-mode perturbation on the front was obtained by deriving an equation for the Laplace transform of the ripple amplitude. Although overlooked at the time Ref. [27] was published, the theory presented there is sufficiently general to permit an examination of the nature of the solution for any set of parameters α, β, and Γ, and thus provides a useful method for analyzing the stability limits of isolated, planar, two-dimensional shock waves. As we have seen, a particularly elegant approach for conducting such an investigation is through an inspection of the poles of the Laplace-transformed ripple amplitude — an expression for which appears in Eq. (12). By analyzing the poles of this Laplace transform, we have demonstrated that two instability classes exist for isolated planar shocks: one in which perturbations grow exponentially in time, and the other in which disturbances are stationary. The necessary criteria for the two classes of instabilities to occur can be stated as $\Gamma < 0$ or $\beta < -1$, and $\Gamma/\alpha^2 > \beta > -1$, respectively. These results agree with those derived by D'yakov [11] and Kontorovich [16] by more laborious means, and serve as an important addendum to our earlier work. In addition, we have shown in this paper that such limits are consistent with several examples obtained by numerically solving the Volterra equation in Eq. (9) for different combinations of the constants α, β, and Γ.

We wish to emphasize that the instability limits derived in this study apply to a particular category of shock waves. This is a consequence of the fact that in arriving at those criteria, several approximations were made, two of which warrant additional comment here. The first is the assumption that the linear sinusoidal deformation to the otherwise-planar shock front occurs suddenly at $\tau = 0$, and we make no inquiry as to the actual physical mechanism (*e.g.*, a constricted boundary channel [18]) by which such a perturbation might arise. As a result, the evolution of the rippled shock can be treated in the context of an initial-value problem with periodic transverse boundary conditions. A second simplifying assumption is that the shock is "isolated," *i.e.*, it is far enough away from the "piston" driving it that the transit time of a sound wave between these two surfaces is much longer than the time of interest for this problem. Such an arrangement is, of course, an idealization and one may ask how the instability limits would change for a more realistic scenario in which acoustic waves impinge upon — and reflect off — a shock front from the compressed fluid region. To a large extent, this remains an open question. We should remark, however, that in the case

of the DK instability, Wouchuk and Cavada [20] have shown that additional eigenmodes of oscillation are excited on the front as the result of a reflecting surface behind it. There is also evidence to suggest that the absolute instability limits appearing in Eq. (2) require modification in the presence of such a surface [12, 33].

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Case	β	Γ	poles	comments
a	-1.100	$+2.553$	$+6.216$, -0.896	positive real pole; exponentially growing solution ($\beta < -1$)
b	$+1.064$	-0.100	$+0.059$, -4.669	positive real pole; exponentially growing solution ($\Gamma < 0$)
c	-0.500	$+2.553$	-1.040, $\pm 2.836\,i$	purely imaginary poles; stationary perturbation ($\Gamma/\alpha^2 > \beta > -1$)
d	$+0.500$	$+2.553$	-1.695, $\pm 1.739\,i$	purely imaginary poles; stationary perturbation ($\Gamma/\alpha^2 > \beta > -1$)
e	$+1.064$	$+2.553$	—	no poles; stable solution (perturbation damps as $\tau^{-3/2}$ as $\tau \to \infty$)

TABLE I: Parameters for the five curves shown in Fig. 4. The value of α^2 in all cases is 2.667.

LIST OF FIGURE CAPTIONS

FIG. 1: Unstable regions of isolated, planar, two-dimensional shock waves in terms of various dimensionless parameters. Definitions are given in the text. Note that the parameters β and Γ are not independent of each other: if $\Gamma < 0$, then $\beta > 0$, and if $\beta < -1$, then $\Gamma > 0$. For $\Gamma > 0$, however, the parameter β may be either positive or negative.

FIG. 2: Plots of $(s^2 + \alpha^2)^{1/2}$ (thick line) and the family of curves given by $-\beta s - \Gamma/s$ (thin solid lines with different values of β and Γ) as a function of the real variable s. Points of intersection on this graph denote the existence of positive real poles of Eq. (12), and thus exponential growing (*i.e.*, unstable) solutions. This can occur in two cases: (i) when $\Gamma < 0$, and concomitantly $\beta > 0$, and (ii) when $\beta < -1$, and concomitantly $\Gamma > 0$.

FIG. 3: Plots of $(\tilde{s}^{\,2} - \alpha^2)^{1/2}$ (thick line) and the family of curves given by $-\beta\tilde{s} + \Gamma/\tilde{s}$ (thin solid lines with different values of β and Γ) as a function of the real variable $\tilde{s}$. Points of intersection on this graph denote the existence of purely imaginary poles of Eq. (12), and thus oscillatory, undamped solutions. This can occur when $\beta > -1$ and the inequality $-\beta\alpha + \Gamma/\alpha > 0$ is satisfied. Note that the latter condition can be rewritten as $\Gamma/\alpha^2 > \beta$.

FIG. 4: Five examples showing the evolution of the amplitude of a linear sinusoidal perturbation on a shock front obtained by numerically solving the Volterra equation in Eq. (9). Each case corresponds to a different set of dimensionless parameters α^2, β, and Γ; see Table I.

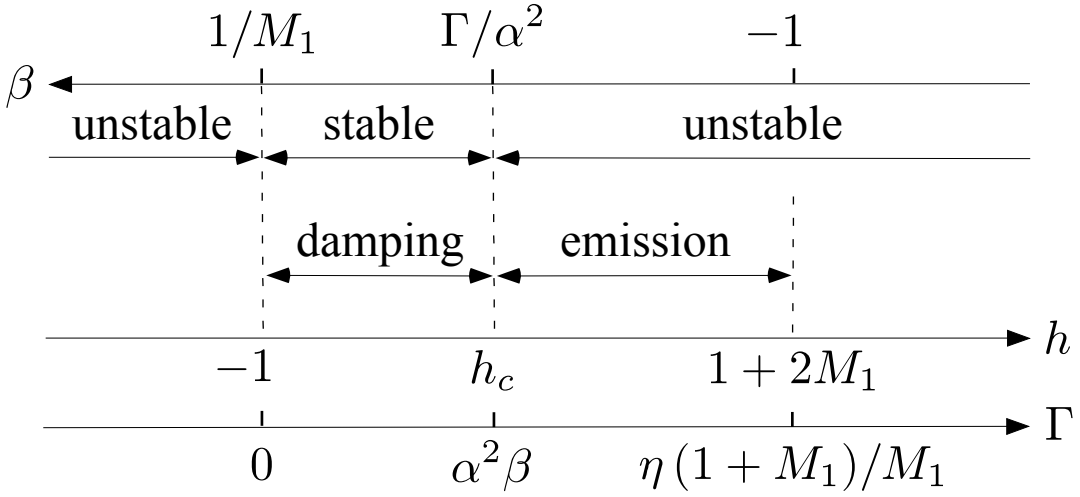
$1/M_1$
Γ/α^2
-1
β
unstable
stable
unstable
damping
emission
h
-1
h_c
$1+2M_1$
Γ
0
$\alpha^2\beta$
$\eta\,(1+M_1)/M_1$

Fig. 1

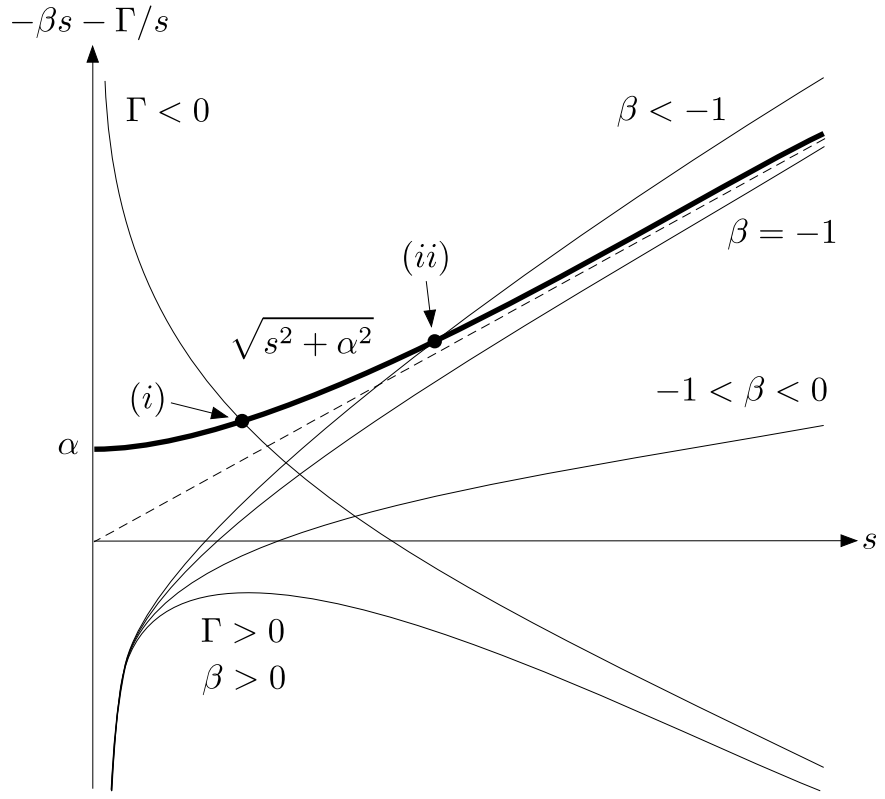
$-\beta s-\Gamma/s$
$\Gamma<0$
$\beta<-1$
$\beta=-1$
(ii)
$\sqrt{s^2+\alpha^2}$
(i)
$-1<\beta<0$
α
s
$\Gamma>0$
$\beta>0$

Fig. 2

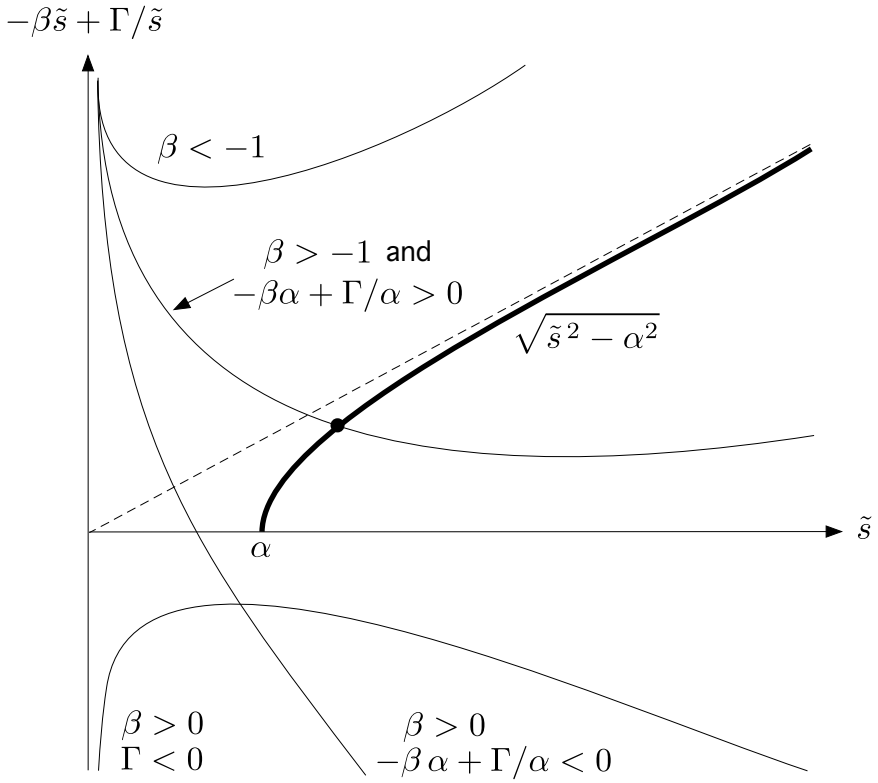
$-\beta\tilde{s}+\Gamma/\tilde{s}$
$\beta<-1$
$\beta>-1$ and
$-\beta\alpha+\Gamma/\alpha>0$
$\sqrt{\tilde{s}^2-\alpha^2}$
$\tilde{s}$
α
$\beta>0$
$\Gamma<0$
$\beta>0$
$-\beta\,\alpha+\Gamma/\alpha<0$

Fig. 3

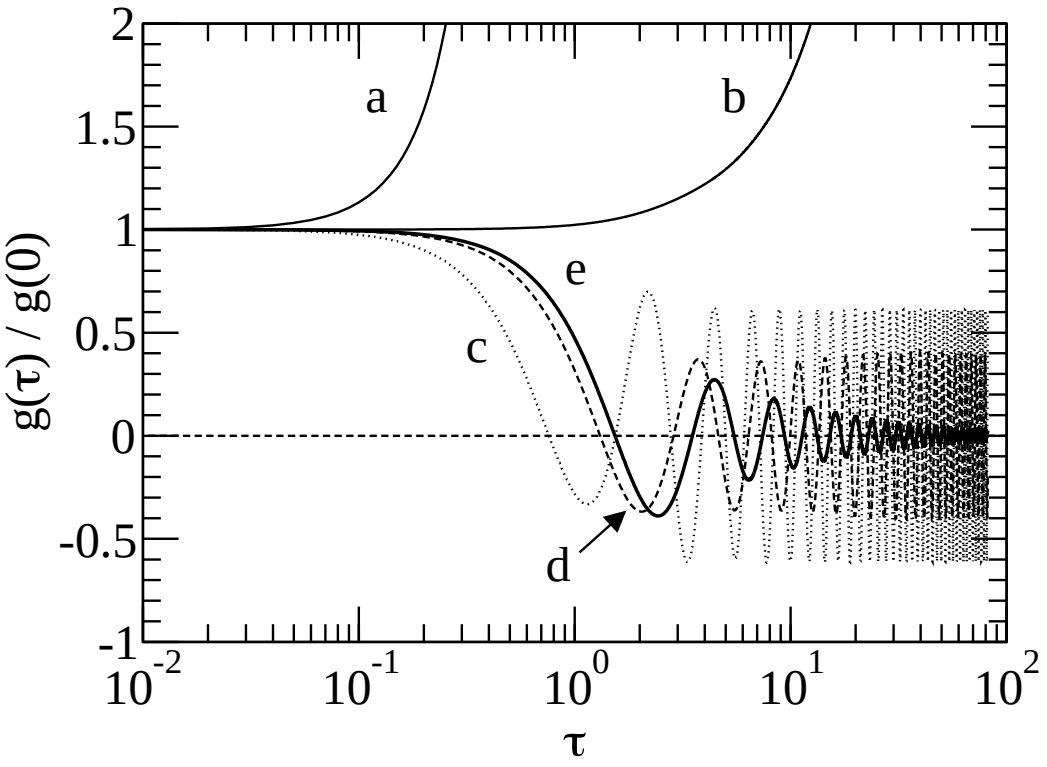
a
b
c
d
e
g(τ) / g(0)
τ
2
1.5
1
0.5
0
-0.5
-1
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

Fig. 4